Activity I: Common Tools of Navigation

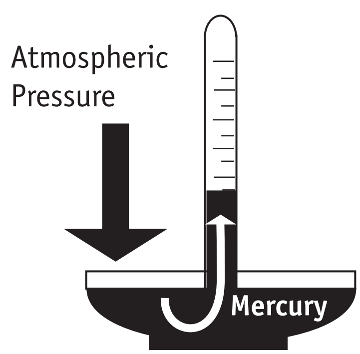
Introduction

Pilots use many different tools for navigation. The use of these tools is based on several scientific and mathematical principles. This section includes just a few of the tools a pilot uses.
They include:

- **Part A. Measuring altitude with pressure gauges**
- **Part B. Measuring position using gravity**
- **Part C. Measuring latitude / longitude or direction with a compass**
- **Part D. Gyroscope-based tools; Gyrocompasses & Attitude Indicators**

Part A - Measuring altitude with pressure gauges

A mercury barometer measures atmospheric pressure. A barometer works in a similar fashion to a thermometer, which also uses mercury. The mercury expands and contracts, based on forces acting upon it. In a barometer, a tube with one open end is placed end-down into a dish filled with mercury. As the atmospheric pressure increases, more and more mercury gets pushed into the tube, such that the tube gets “marked” at higher and higher levels with mercury.



Atmospheric pressure is a function of the height of the mercury in the tube (h), the density of mercury (ρ), and acceleration due to gravity (g).

$$P_{atm} = \rho g h$$

Pressure will vary due to weather conditions, altitude, and acceleration due to gravity (usually negligible).

1. Although most barometers are made with mercury, other liquids can be used. For instance, red wine was used often for early barometers.

   a) What characteristics would you want your barometer liquid to have?

   b) Calculate the range in height in meters for barometers responding to a range of 1.0×10^4 to 1.0×10^6 Pascals, for the following liquids with shown characteristics. 1 Pascal = $kg/(s^2m)$ or 1 N/m^2

	Density (kg/m^3)	**Boiling Point (° C)**	**Melting Point (° C)**	**Molar Mass (g/mol)**
Mercury	16.3×10^3	357	-38.87	200.59
Iodine	4.94×10^3	184.35	113.5	126.90
Whole Blood	1.06×10^3	about 100	about 0*	
Water	1.0×10^3	100	0	
Iron	7.8×10^3	2750	1535	55.85

*Blood has clotting agents that facilitate "solidifying" process (coagulation) at any temperature.

   c) Which is the best liquid to use, and why?

Here is a link to a site which will show you how to build your own barometer!
http://www.met-office.gov.uk/education/curriculum/es15.html

2. Fill in the following chart with atmospheric pressure values or height of the mercury in a barometer. Assume the following are constants:
   $\rho = 1.35955 \times 10^4$ kg/m^3 (at 0 degrees C) , g = 9.80665 m/s^2
   ρ is "rho", the standard symbol used for the density of air.

	What is the Ambient Pressure:	p_{atm}	h(m)
1.	During a passing storm on Earth	9.7×10^4	
2.	On the surface of Venus		68.25
3.	At the bottom of the deepest ocean trench on Earth	1.1×10^8	
4.	At sea level on Earth		0.76
5.	Blood Pressure (normal)	1.6×10^4	

3. Air pressure may not change significantly in two places that are close to each other, but pressure can vary a great deal across great distances, especially vertical distances. In addition, the density of air (ρ) can differ a great deal depending on altitude.

   It is assumed that density is proportional to pressure (which would be true if temperatures of air remained the same at different altitudes, according to the ideal gas law). With this assumption, we can model how pressure changes with altitude. Neglecting variation in g with altitude (assume g = 9.80 m/s^2), we can find the pressure of air (p) at any altitude above sea level (y).

   In the following equation, pressure at sea level is p_0, and a is a constant (8.55 km) that reflects the change in altitude over which the pressure drops by a factor of e.

$$p = p_0 e^{-y/a}$$

a) Assume $p_0 = 1.01 \times 10^5$ Pascals. Solve for pressure in atm and Pascals.
1 atm = 1.013×10^5 Pascals.

	Location	Altitude (y) in km	Pressure (p) in Pascals
1.	Top of Sears Tower (Chicago)	about 0.9	
2.	Mt. Hvannadalshnukur (Iceland)	2.1	
3.	Mt. Victoria (Papua New Guinea)	4	
4.	Mt. Acancagua (Argentina)	7	
5.	Mt. Everest (Nepal)	about 9.6	
6.		10	
7.		10.5	
8.	Typical Cruising Altitude of Commercial Jet	11	
9.		15	
10.		20	
11.		30	

b) Graph the results from the previous chart and determine the equation for the line described by the function on a coordinate grid comparing altitude (x-axis) to pressure (y-axis).

c) Pilots can use air pressure to determine altitude. Rewrite your equation with pressure as x and altitude as y.

4. Airline crew modify the pressure and temperature of the air in the airplane cabin in order to make airline passengers more comfortable. The system that they typically use allows pilots to set a pressurization controller to a comfortable pressure. In response, air slowly leaks in and out of the cabin until the desired pressure is achieved (much like how a heater works). Usually the pressure in the cabin is a little lower than the pressure of air when the airplane took off. This is because the airplane is not completely airtight, so the air pressure will decrease as the airplane flies in a low-pressure region (high altitude). This is why your ears often pop after takeoff or during descent.

a) When at cruising altitude, is the pressure inside the airplane more or less than the pressure outside the airplane?

b) If a window were to open, would air rush into or out of the airplane, in order to equalize pressure?

Part B - Measuring position using gravity

In general, the Earth is a spherical body, whose density depends on the radial distance from its center. The magnitude of gravitational force (G) acting on any given piece of matter can be calculated. The following equation uses this general model, the mass of the matter (m), the distance of the matter from the center of the Earth (r), and the mass of the earth (M_E), to determine the magnitude of gravitational force.

$$F = GM_E m/r^2$$

We can combine this equation with the equation for gravitational force, derived with Newton's Second Law, using the relationship between mass of the object and free-fall acceleration due only to gravitational pull (g_0).

$$F = mg_0$$

Thus

$$mg_0 = GM_E m/r^2$$

and in a more simplified form

$$g_0 = GM_E/r^2$$

This equation can be used to compare gravity at different distances from the center of the earth. This, of course, takes altitude into account!

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1. Use the equation on the previous page to determine how gravity changes with altitude. Assume that the mass of the Earth is 5.97×10^{24} kg, the mass of the object is 10 kg, and the radius of the Earth is 6.37×10^{6}m. $G = 6.67259 \times 10^{-11}$ Nm^2/kg^2

	Location	Altitude (km)	Radius (r_{Earth} + altitude)	g_0
1.	Sea level	0		
2.	Mt. Kazbek (former Soviet Union)	5		
3.	K2 (Pakistan) + 1.4 km	10		
4.	Typical Cruising Altitude of Commercial Jet	11		
5.		50		
6.		100		
7.		400		
8.	Communications Satellite orbit	35,700		
9.	Orbit of the Moon (but not on the Moon)	38,000		

2. Draw a graph showing how altitude and acceleration due to gravity are related.

3. What is the acceleration due to gravity at your house / in your town?

altitude = ____________ g_0 = ______________________

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When the previous equation is investigated more, one finds that while the afore-mentioned equations may be generalizing a trend, there are many exceptions. This is largely due to problems with the first generalization we made; the Earth is not a perfectly spherical body whose density depends on its radial distance from its center. In fact, the Earth is not a sphere - it is actually ellipsoid, flattening at poles and bulging at the equator. The equatorial radius is 21 km larger than its polar radius. Because of this, if one measures free-fall acceleration for the same object at the north pole, acceleration due to gravity will be more than that measured at the equator.

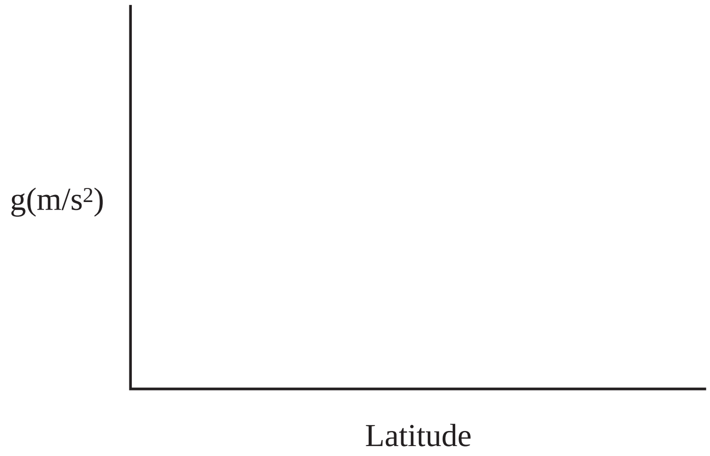
In addition, the Earth's crust is not uniform. There are not only variations in density everywhere, but the crust's depth varies in places, as well.

Lastly, the Earth is rotating, which influences the acceleration of an object. The mass of an object will not vary, regardless of its position on Earth, but g_0 and g may differ, depending on distance from the equator. This influences the final centripetal acceleration of the object.

4. What follows is a tablethat estimates the measure of gravitational acceleration (g) at different latitudes. From these values, draw a graph and estimate an equation reflecting the relationship between gravity and latitude.

Latitude (degrees)	g(m/s^2)
0	9.780
10	9.782
20	9.787
30	9.793
40	9.802
50	9.811
60	9.821
70	9.829
80	9.832
90	9.835

a) graph (use a ruler or draw yours on graph paper, to be extremely accurate!):



b) approximate linear equation:

____

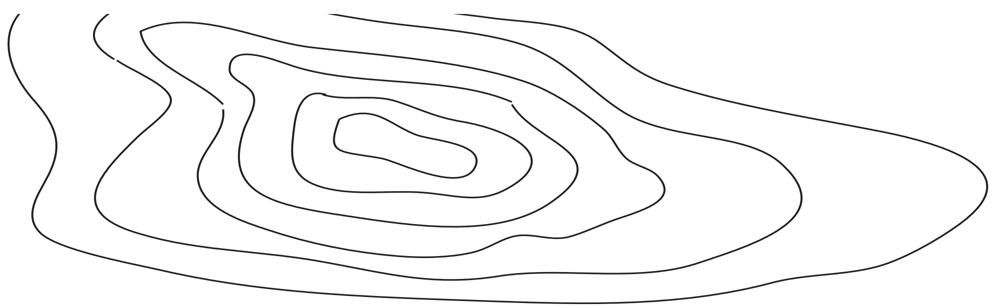
c) Plot your approximate equation in another color, on the coordinate grid above.

d) Is the pattern seen in the graph best explained by a linear equation? Explain. If not, describe what kind of equation it looks like.

5. We can visualize geological and topographic features using gravity. When we gather information about gravitational force in an area, we can use it to create a picture that is very similar to those created by lines of elevation on a topographical map. For instance, if oil were in a certain region below the ground, the gravitational force directly above the oil would be less than what it typically is.

The map we would find, that might indicate to us that there is an oil reserve, might look like this. At the center of the oil pool (the smallest shape), the gravitational force would be the least, and would gradually or quickly (depending on proximity of different gravitational levels, depicted by the contours) increase.

Top view:



Side view to show depth:

In addition to detecting oil, buried cities or archaeological sites, buried caverns or underground facilities, active volcanoes, faults, aquifers, and other kinds of mineral deposits can be found using differences in gravitational force.

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Armed with equipment that can measure gravity, any airplane could assess its position relative to different features on the Earth's surface, just as it would determine position based on proximity to structures of specific altitudes. Although these methods are usually not used due to a variety of factors (imprecise measuring tools, altering Earth features, for instance, in the oceans and aquifers, and minor changes in gravitational force over large distances), such information could prove useful for future navigation ventures, particularly in areas where compasses are of little use.

In fact, measuring gravity has been used on Mars missions, in order to determine depth of the crust and existence of volcanoes, among other things.

Part C - Measuring latitude, longitude or direction with a compass

The simplest compass is a suspended needle that points, via magnetism, to magnetic north. On an airplane, instead of saying “N” or “North,” the compass reads “360°.” For South, it reads “180°,” for East it reads “90°,” and for West it reads “270°.” Determining direction is just one component of navigation; often more important is being able to determine position.

We evaluate position by determining how a point occurs relative to the equator (zero latitude) and the zero meridian (zero longitude). Position relative to these lines is recorded in terms of degrees, minutes and tenths of minutes. A minute is a sixtieth of a degree, just as a minute is a sixtieth of an hour. For instance, Birmingham Airport is at N5227.4/W00145.2 which means it is North of the equator by 52 degrees and 27.4 minutes, and West of the zero meridian by 1 degree 45.2 minutes.

1. Describe the positions for the airports listed below. Check your work and locate the airports on a globe or map that shows latitude and longitude.

	Airport	Position	Description
1.	Miami Airport, FL, USA	N2547.1 W08016.6	
2.	Perth Airport, Australia	S3156.8 E11557.5	
3.	Beirut, Lebanon	N3353.6 E03528.7	
4.	Jakarta, Indonesia	S0619.2 E10817.1	

2. Decide which major city's airport is located at each given position.

	Position	Airport
1.	N0117.1 E10349.3	
2.	N5215.1 E02059.1	
3.	N5043.6 W00003.4	
4.	S0121.8 E03656.7	

3. Approximate in direction, degrees and minutes, the location of the following airports.

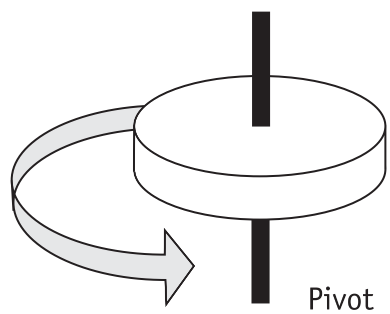
	Airport	Position
1.	Portland Airport, OR, USA	
2.	Orly Airport, Paris, France	
3.	O'Hare Airport, Chicago, IL, USA	
4.	Tokyo Airport, Tokyo, Japan	

4. Work in pairs. One person provides either (a) location of airport OR (b) name of airport. The other person provides the missing information. (You choose your values as you go along.)

Part D - Gyroscope-based tools; Gyrocompasses & Attitude Indicators

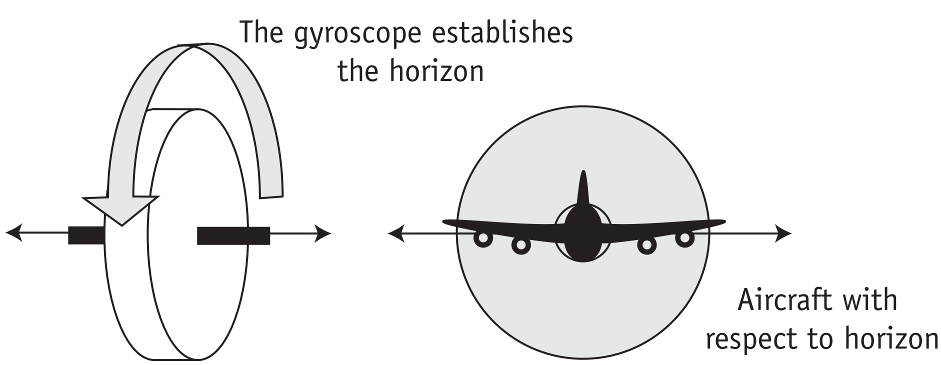
How does a gyroscope work?

A gyroscope contains an axis with two tips, one of which is a pivot. Perpendicular to the axis is a wheel. Once the wheel is set spinning, the gyroscope resists forces such as gravity that may act along its axis (it experiences gyroscopic inertia). In this way, the gyroscope balances and maintains its spinning position eternally, provided that there is no friction where it pivots and no forces act upon the wheel (like wind) causing the wheel to fall.



A gyrocompass is made up of a gyroscope that keeps pointing North once it has been set in that direction.

Attitude Indicator (AI or "artificial horizon") has a gyroscope inside to determine whether the flight plane is straight and level. The attitude indicates where the nose and the wings of the aircraft are with respect to the horizon. The gyroscope's axis is East-to-West, forming a horizon. It maintains this horizon, regardless of gravity and other forces when the airplane turns or banks. The airplane has a separate part that shows the position of its nose and wings.



It is important for directional gyroscopes to reset themselves as directions change. In other words, as the magnetic north changes, the gyroscope needs to reset itself so that its horizon is correct (Remember, Earth is not flat, but spherical!). How does this work? Our gyroscope is attached to a weight that acts as a pendulum, and can tilt the axis using gravity. When gyroscopes alter position to take into account direction, they are called gyrocompasses.

1. Does the gyrocompass point to true north or magnetic north?

2. What might be some benefits of having a device point to true north rather than magnetic north?

3. Although a pilot could determine nose and wing position relative to the horizon by vision, the attitude is especially important during the nighttime or poor weather, when it is difficult to see the horizon. When might the attitude also be important?

Inertial Reference System (IRS) is another navigational tool that measures position with respect to latitude, longitude, and horizon, as well as ground speed. It is made up of laser gyroscopes and accelerometers. The gyroscopes provide information to the pilot about attitude and heading signals. The accelerometers keep tabs on ground speed in order to measure distances traveled, and thus position.